1. Nucleation and Growth by Diffusion

Planck Eq.

Fokker-Thermodynamic

Freundlich Eq.

Ostwald-Zeldovich-Nucleation and Growth of Colloidal Cluster by Diffusion under Ostwald-Freundlich Boundary Condition

2. Thermodynamic and Kinetic Critical Radius

Thermodynamic Critical Radius $R_c$

$$\frac{\partial \ln(n(t))}{\partial t} - \beta \frac{\partial \Delta G(n)}{\partial n} = \frac{\partial n(t)}{\partial n}$$

Zeldovich Relation

$$\frac{dn}{dt} = -\beta n \frac{\partial \Delta G(n)}{\partial n} = -\beta n \mu(n) = \nu(n) \left( \frac{c(n)}{c_0} \right)$$

Ostwald-Freundlich Eq.

$$c_0(R) = c_0 \exp \left[ -\beta \mu(R) \left( \frac{1}{R} - \frac{1}{R_s} \right) \right]$$

Thermodynamic Growth Eq.

$$\frac{dR}{dt} = \beta \mu(R) \left( \frac{V_n(R)}{4D^2} \right) \left( \frac{1}{R} - \frac{1}{R_s} \right)$$

Kinetic Critical Radius $R_k$

$$\frac{dR}{dt} = \nu \left( \frac{c(R)}{c_0} \right)$$

Diffusional Growth Eq.

$$\frac{dR}{dt} = \beta \mu(R) \left( \frac{V_n(R)}{4D^2} \right) \left( \frac{1}{R} - \frac{1}{R_s} \right)$$

Kinetic

Thermodynamic

3. Coupling of Surface Processes

3.1 Core-shell composite nucleus

Diffusion Flux at $R$

$$J_s = D \left( \frac{c_s}{R} - \frac{c(R)}{R} \right) - D_{a\eta} \left( \frac{c_s}{R} - \frac{c(R)}{R} \right)$$

Diffusion Flux at $R_s$

$$J_{s\eta} = D \left( \frac{c_s}{R_s} - \frac{c(R_s)}{R_s} \right) - D_{a\eta} \left( \frac{c_s}{R_s} - \frac{c(R_s)}{R_s} \right)$$

Conservation of Material

$$\frac{dN}{dt} = 4\pi R^2 J_s$$

$$N = \int_0^R 4\pi (r^2) dr$$

Evolution Eq. for $dN/\partial t$, $dR_s/\partial t$

3.2 Surface Chemical Reaction

$$\frac{dR}{dt} = \nu \frac{Dc_0}{D + k_f + R} \left( \frac{1}{k_f + k_b} \right) \frac{c_0}{c_0}$$

$$\frac{dR}{dt} = \beta \mu(R) \left( \frac{V_n(R)}{4D^2} \right) \left( \frac{1}{R} - \frac{1}{R_s} \right)$$

Diffusion Controlled

$$\alpha \left( \frac{1}{R} \right) \frac{1}{R_s}$$

k$_b$ << D Reaction Controlled

$$\alpha \left( \frac{1}{R} \right) \frac{1}{R_s}$$

4. Conclusion

We considered the connection between the thermodynamic Zeldovich relation and the diffusional growth equation. A thermodynamic critical radius and a kinetic critical radius can be defined. They become identical if the Ostwald-Freundlich boundary condition is satisfied. This result will not be affected even if complex surface processes exist. Even when the nucleus is surrounded by a surface layer, the diffusion controlled growth has a possibility to lead to a mono-dispersed size distribution.

5. References